Niveau(x) d'enseignement: 3 Numéro EDUQ : 8370133
Centre d'intérêt: 841

Lunkenbein, Dieter, 1937-.

Deltagones = Deltagons / [D. Lunkenbein ; réd., J. Roy ; dessins et trad. anglaise, P. Allard]. -- Sherbrooke : Université de Sherbrooke, Département de mathématiques et d'informatique, [1981?].
[9, 9] p. : ill., ; 28 cm.

Titres des couv. Texte en français et en anglais tête-bêche.

Cote Dewey: 372.7/L963d Cote LC: QA/11

Descripteur(s): *Mathématiques *Perception visuelle *Jeu éducatif *Guide pédagogique *Primaire (Niveau)

Identificateur(s) de sujet: *Géométrie *Deltagone *Structuration des connaissances

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Le document présente un matériel d'apprentissage constitué de formes géométriques planes, lequel sert de support concret à des activités de nature arithmétique et géométrique. De nombreuses relations entre ces deux domaines mathématiques peuvent être illustrées à l'aide de ce matériel: ainsi des relations insoupçonnées entres les nombres sont mises en lumière par l'étude de certaines propriétés quantitatives des deltagones, ou bien des données numériques déjà connues acquièrent une signification nouvelle en étant concrétisées par ces formes géométriques.

En plus des propriétés quantitatives des deltagones, le document s'attarde aux propriétés qualitatives de ces formes polygonales ainsi qu'aux façons permettant d'engendrer ou de varier de telles formes. Les activités suggérées mettent l'accent sur l'émergence de processus plutôt que sur l'acquisition de connaissances précises; à ce titre, elles accordent une place particulière aux créations et aux stratégies personnelles de l'élève.

Type de document: d
Financement: Bénéficiaire - Université de Sherbrooke
Organisme(s) source(s) - MEQ
Programme(s) de subvention - FCAC
Auteur de l'analyse: ar

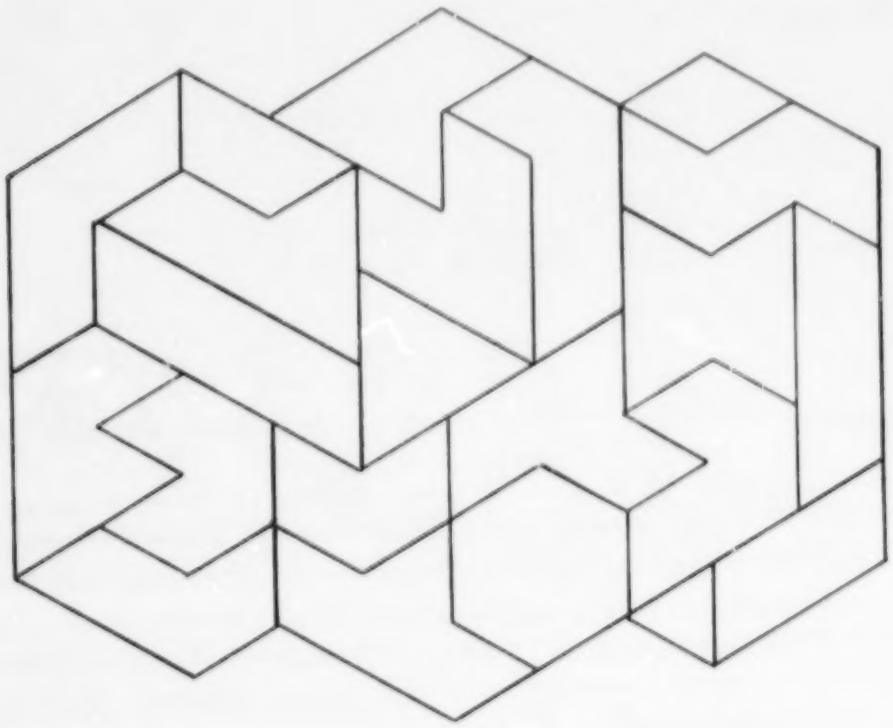
E D U Q 8370133

DELTAGONES

DESCRIPTION GÉNÉRALE

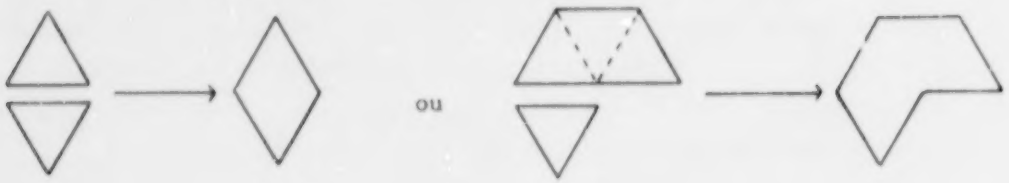
Ce matériel d'apprentissage, constitué de formes géométriques planes, sert de support concret à des activités de nature arithmétique et géométrique. De nombreuses interrelations entre ces deux domaines de la mathématique peuvent être illustrées à l'aide de ce matériel. Ainsi des relations insoupçonnées entre les nombres seront mises en lumière par l'étude de certaines propriétés quantitatives des deltagones, ou bien, des données numériques déjà connues acquiéreront une signification nouvelle en étant concrétisées par ces formes géométriques.

En plus des propriétés quantitatives des deltagones, on s'intéressera également aux propriétés qualitatives de ces formes polygonales ainsi qu'aux façons permettant d'engendrer ou de varier de telles formes. Les activités suggérées mettent l'accent sur l'émergence de processus plutôt que l'acquisition de connaissances précises et, à ce titre, accordent une place particulière aux créations et aux stratégies personnelles de l'étudiant.



DESCRIPTION PARTICULIÈRE

Des triangles équilatéraux congrus peuvent être assemblés côté à côté de façon à former des polygones variés. Par exemple:



Puisque le triangle équilatéral est l'élément de base de ces polygones, on appellera ceux-ci des deltagones en raison de la forme de la lettre grecque delta majuscule (Δ).

Les polygones ainsi formés peuvent donc être caractérisés par le nombre de triangles élémentaires dont ils sont composés. En cherchant à découvrir tous les deltagones possibles comportant un, deux, trois, quatre, cinq, six triangles, nous obtenons les deltagones suivants:

un triangle:

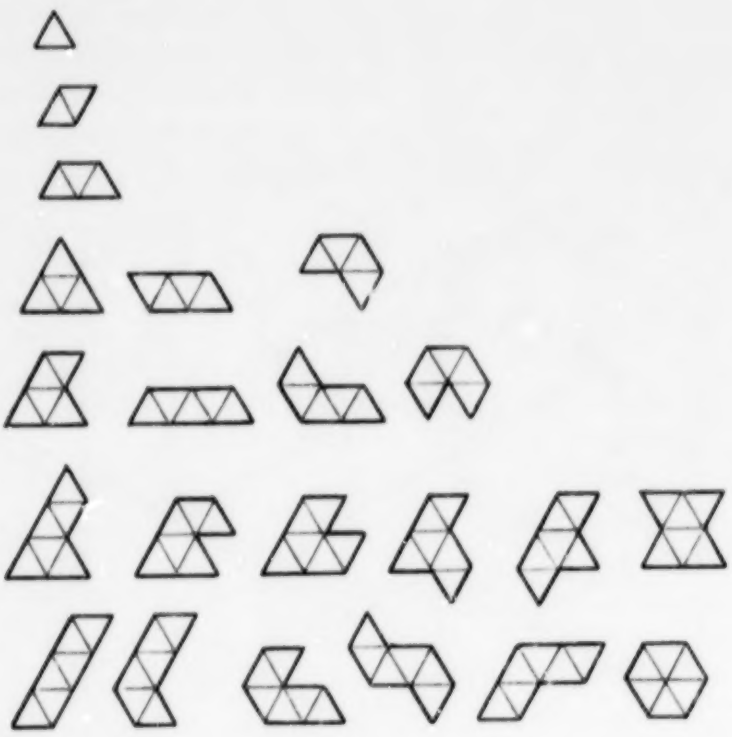
deux triangles:

trois triangles:

quatre triangles:

cinq triangles:

six triangles:

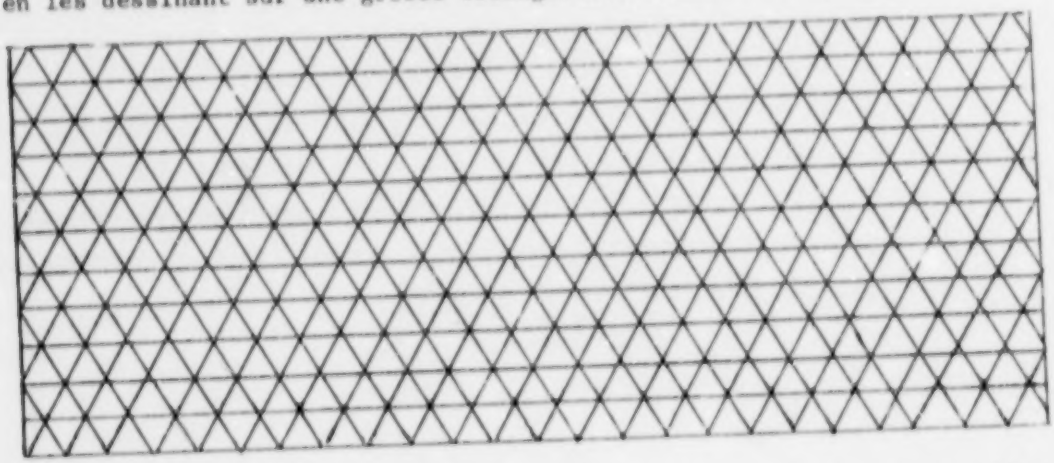


Le présent matériel comporte 30 triangles élémentaires (Δ), 20 losanges (◇), 10 trapèzes (⏢) et un représentant de chacun des deltagones composés de quatre, cinq et six triangles.

SUGGESTION D'ACTIVITES

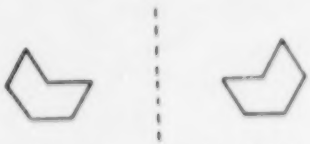
A <u>Création et discrimination de deltagones</u>

En composant les triangles équilatéraux existant dans le matériel, on peut créer plusieurs deltagones que l'on pourra ensuite prendre en note en les dessinant sur une grille triangulaire comme celle qui suit:

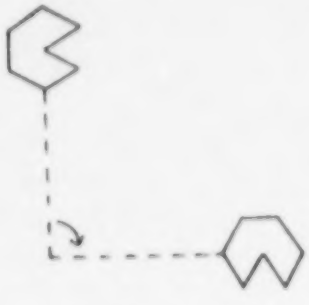


En cherchant à créer des deltagones tous différents, les étudiants s'interrogeront sur la façon de déterminer si deux deltagones sont identiques ou non. Il s'avérera alors intéressant d'identifier les formes pouvant être superposées par translation, réflexion symétrique ou rotation.

translation



réflexion symétrique



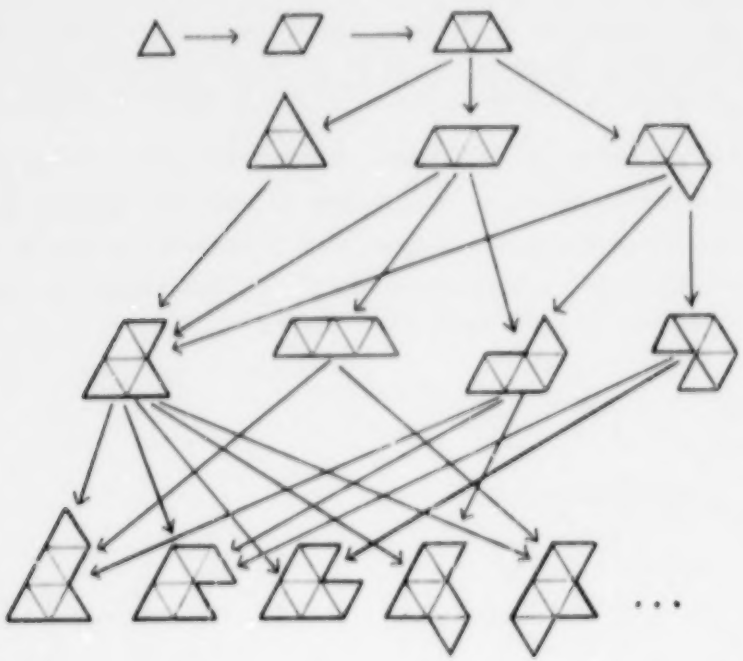
rotation

La description des différents deltagones qui ont été créés fera découvrir certaines propriétés de ces formes géométriques telles que le nombre de côtés ou de sommets, la ressemblance avec des formes familières (triangles, trapèzes, parallélogrammes), le nombre de triangles élémentaires dont ils sont composés, la convexité, etc.

B Recherche de deltagones composés d'un nombre déterminé de triangles élémentaires

La question qui suit pourra amorcer cette activité: "Combien existe-t-il de deltagones différents composés d'un triangle, deux triangles, trois triangles, quatre triangles, cinq triangles, six triangles?".

La recherche de tous ces deltagones suscitera une réflexion sur la façon de construire ces formes géométriques et nécessitera l'utilisation d'un procédé plus systématique permettant d'être sûr des résultats obtenus. L'élaboration de stratégies concernant l'engendrement des formes et les façons de modifier celles-ci est ainsi une des habiletés que cette activité permet de développer. Le procédé illustré sur le schéma suivant consiste à ajouter un triangle élémentaire de toutes les façons possibles à un deltagone plus petit déjà construit.

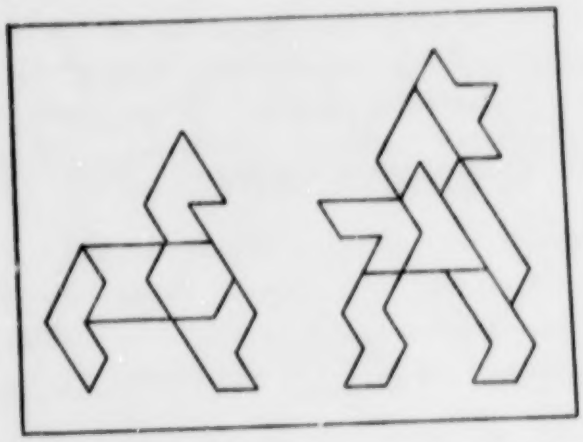


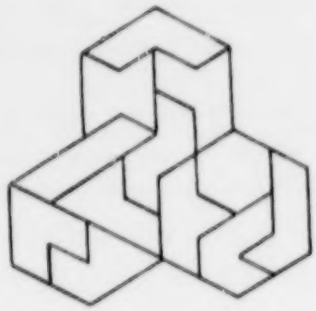
Cette activité oriente les étudiants vers la classification des deltagones selon le nombre de triangles élémentaires dont ils sont composés, c'est-à-dire selon l'aire, la grandeur de ces formes géométriques et favorise, de ce fait, l'exploration des variations possibles de formes de même grandeur. Ces formes de même grandeur pourront par la suite être caractérisées par d'autres propriétés telles que le nombre de côtés, le périmètre, la convexité, le nombre de symétries axiales, etc.

C Casse-tête de deltagones

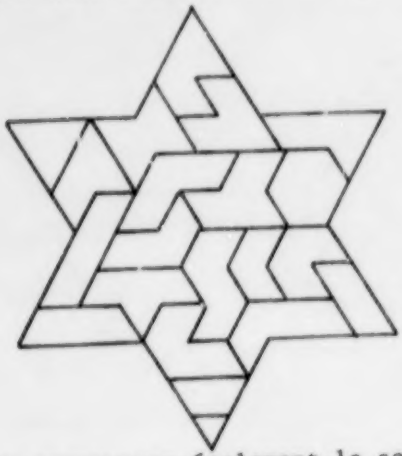
Par les activités de décomposition et de recomposition qu'ils comportent, les casse-tête viennent stimuler l'imagination spatiale et favoriser de façon indirecte l'exploration des formes géométriques. Ce genre d'activité à caractère ludique utilise les ressources créatrices des étudiants qui pourront tout aussi bien composer des scènes fantaisistes que des figures de style plus géométrique. Nous en donnons quelques exemples.

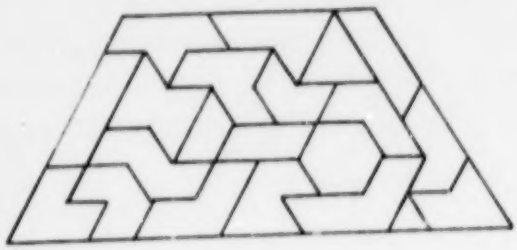
Cette scène fantaisiste composée avec les douze hexadeltagones peut être transformée en cette forme géométrique à droite tirée d'un article de Martin Gardner[1].





Cette étoile constituée de tous les deltagones, à l'exception du losange (▱), peut être transformée en trapèze.





On remarquera également le casse-tête illustré sur la page-titre de cette brochure composé de tous les deltagones.

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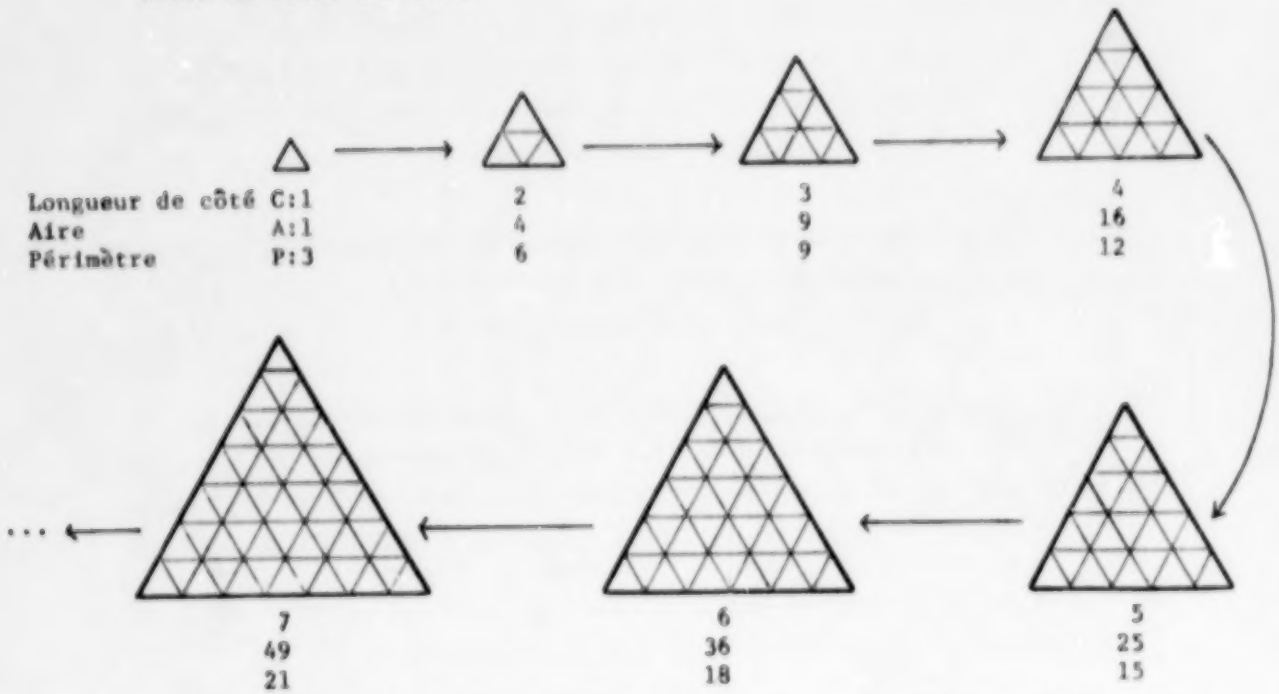
1. Martin Gardner: On polyiamands: shapes that are made of equilateral triangles, Scientific American, Dec. 1964.

D Exploration des deltagones triangulaires

Cette activité pourra débuter par la création de plusieurs deltagones triangulaires différents:

On remarquera vite que ces deltagones ne peuvent être différenciés qu'au moyen de leur grandeur respective puisqu'ils sont tous de même forme. Chaque deltagone triangulaire peut en effet être déterminé par une des trois valeurs quantitatives suivantes: p: longueur d'un côté, p^2: aire (nombre de triangles élémentaires dont il est composé) et 3p: périmètre.

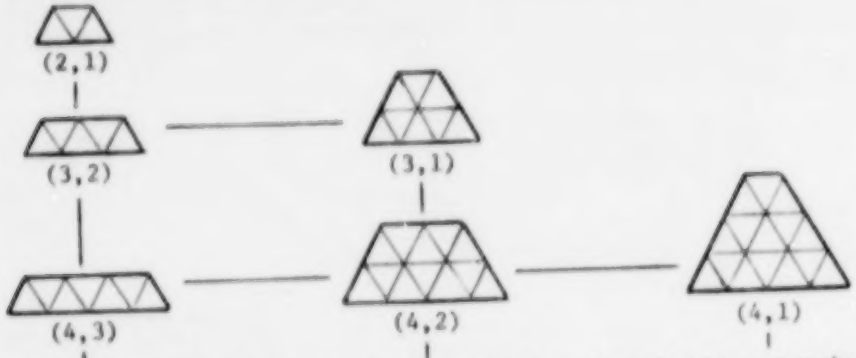
En recherchant tous les deltagones triangulaires possibles, on obtiendra alors la série suivante:



Les trois séries de nombres rattachées à cette suite d'objets géométriques montrent qu'il existe autant de deltagones triangulaires qu'il y a de nombres naturels (p>0), ou de nombres carrés (p^2>0), ou de multiples de trois (3p>0).

E Exploration des deltagones trapèzes

On recherchera tout d'abord les différents deltagones trapèzes possibles:

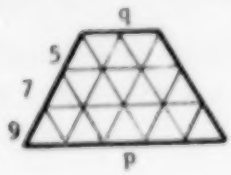


Chacun de ces deltagones trapèzes peut être décrit par une paire ordonnée de nombres naturels (p, q), p>q>0, représentant la longueur des deux côtés parallèles. On observera que les côtés non parallèles sont d'égale longueur, soit (p - q). Avec ces données, il sera alors facile d'élaborer une formule permettant de calculer le périmètre des deltagones trapèzes.

Il existe plusieurs façons par lesquelles on peut aborder le calcul d'aire des deltagones trapèzes. Une de ces façons part du fait que tout deltagone trapèze peut être considéré comme le restant d'un deltagone triangulaire p duquel on a enlevé le deltagone triangulaire q.

Par conséquent, l'aire d'un deltagone trapèze (p, q) peut être calculée comme la différence $p^2 - q^2$ des aires des deltagones triangulaires p et q.

Une autre façon de calculer l'aire d'un deltagone trapèze a son origine dans l'observation suivante: tout deltagone trapèze est constitué d'une série de trapèzes élémentaires successifs et chacun de ces trapèzes élémentaires est composé d'un nombre impair de triangles équilatéraux.

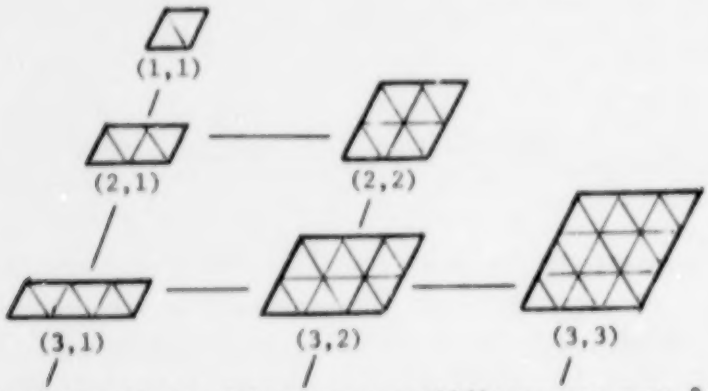


La somme de cette suite de nombres impairs successifs, allant de $2q + 1$ à $2p - 1$, donnera l'aire du deltagone trapèze.

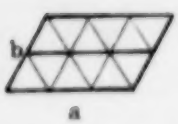
Certaines relations entre les nombres sont ainsi mises en lumière par ces deux façons d'aborder l'aire des deltagones trapèzes, comme par exemple: la différence entre deux nombres carrés peut toujours être exprimée par la somme d'une série de nombres impairs successifs, la différence entre deux nombres carrés successifs est toujours un nombre impair, etc.

F Exploration des deltagones parallélogrammes

Les deltagones parallélogrammes peuvent être définis par une paire ordonnée de nombres naturels (a, b) a≥b>0, représentant la longueur respective de leurs deux côtés non parallèles.



Le calcul d'aire de ces deltagones parallélogrammes peut être abordé de plusieurs façons. Comme on peut le remarquer, chacun de ces deltagones parallélogrammes est constitué d'une série de parallélogrammes élémentaires, et chacun de ces parallélogrammes élémentaires est composé de 2a triangles élémentaires.



Puisque l'on retrouve un nombre b de parallélogrammes élémentaires dans un deltagone parallélogramme, l'aire de ce dernier sera par conséquent égale à 2ab (triangles élémentaires).

On peut également considérer tout deltagone parallélogramme (a, b) comme le restant d'un deltagone triangulaire (a + b) duquel on a enlevé les triangles a et b.

Par conséquent, l'aire d'un deltagone parallélogramme (a, b) peut être calculée par: $(a + b)^2 - a^2 - b^2$.

En mettant en relation les deux façons de calculer l'aire des deltagones parallélogrammes, on constate que $2ab = (a + b)^2 - a^2 - b^2$, c'est-à-dire que $(a + b)^2 = a^2 + 2ab + b^2$, et nous trouvons ainsi une représentation "deltagonale" de la formule binômiale.

C Exploration des deltagones semblables

On commencera cette activité par la création de deltagones semblables. Ces deltagones de même forme peuvent seulement varier par leur grandeur. En voici un exemple:

En observant ces deltagones, on s'aperçoit que pour être semblables, deux deltagones doivent avoir les mêmes proportions. Par exemple, des côtés qui sont de même longueur dans une figure doivent également l'être dans la figure semblable; si un côté est deux fois plus long qu'un autre dans une figure, ce doit également être le cas dans la figure semblable. Ainsi, dans l'exemple illustré ci-haut, on trouve que la proportion entre la base du bateau et un de ses côtés est 2:1 pour la première figure, 4:2 pour la seconde et 6:3 pour la troisième. Différents deltagones semblables peuvent donc être caractérisés par des expressions proportionnelles différentes mais équivalentes. La relation "même forme" dans le domaine des figures géométriques correspond alors à la relation "même proportion" ou "même ratio" dans l'univers des nombres.

Des deltagones semblables peuvent être ordonnés comme des expressions proportionnelles équivalentes, ou des fractions équivalentes, selon la grandeur de leurs composantes. En établissant de telles séries de deltagones où l'on double, triple, etc., la longueur des côtés correspondants, on peut observer l'effet de ces opérations sur l'aire. On réalisera alors que si la longueur des côtés d'un deltagone est n-uplée, l'aire de ce deltagone sera n^2-uplée.

L'exploration des deltagones semblables permet donc de concrétiser les notions d'expressions proportionnelles équivalentes, de fractions équivalentes et fait ressortir certains liens entre mesure de longueur et mesure d'aire.

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1. Gardner, M.: On polyiamands: shapes that are made of equilateral triangles, Scientific American, December 1964.

2. Lunkenbein, D.: Deltagone: Formen und Zahlen, Mathematiklehrer, 1-1982 und 2-1982.

3. Lunkenbein, D.: Deltagons: An Exploration of Shapes and Numbers, Université de Sherbrooke, Département de mathématiques et d'informatique, 1982.

Auteur D. Lunkenbein
Rédaction J. Roy
Dessins (et traduction anglaise) P. Allard

Réalisé dans le cadre d'un projet de recherche en didactique de la mathématique subventionné par le Ministère de l'Education du Québec (F.C.A.C., EQ-646); Département de mathématiques et d'informatique, Université de Sherbrooke, Sherbrooke, Québec, Canada, J1K 2R1.

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1. Gardner, M.: On polyiamands: shapes that are made of equilateral triangles, Scientific American, December 1964.

2. Lunkenbein, D.: Deltagone: Formen und Zahlen, Mathematiklehrer, 1-1982 und 2-1982.

3. Lunkenbein, D.: Deltagons; An Exploration of Shapes and Numbers, Université de Sherbrooke, Département de mathématique et d'informatique, 1982.

Author — D. Lunkenbein
Editor — J. Roy
Drawings (and English translation) — P. Allard

Produced within a research project on the didactic of mathematics subsidized by the Ministère de l'Education du Québec, (F.C.A.C., EQ-646); Département de mathématiques et d'informatique, Université de Sherbrooke, Sherbrooke, Québec, Canada, J1K 2R1.

G Exploring similar deltagons

This activity begins by the creation of similar deltagons. Such deltagons may vary in size only. Here is an example:

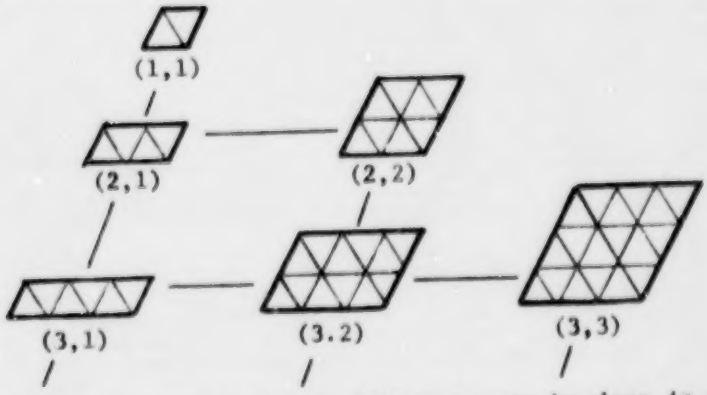
By observing these deltagons, we see that to be similar, two deltagons must have identical proportions. In other words, sides that are of the same length in one figure must be of equal length in the similar figure; if one side is twice as long as another one in one figure, it must also be the case in the similar figure. Therefore, in the above drawings, we find that the ratio between the basis of the boat and one of its side is 2:1 for the first figure, 4:2 for the second and 6:3 for the third. Different similar deltagons can thus be characterized by proportional expressions that are different but nevertheless equivalent. The relation "same shape" within the realm of geometrical figures corresponds, therefore, to the relation "same proportion" or "same ratio" in the world of numbers.

Similar deltagons can be ordered as equivalent proportional expressions or equivalent fractions according to the size of their constituents. By forming such series of deltagons through the doubling, the tripling, etc., of their corresponding sides, we can observe the effect of those operations on the area. We will then realize that whenever the length of the sides of a deltagon is n-upled, the area of the deltagon is n^2-upled.

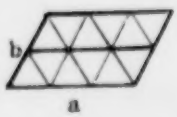
This exploration of similar deltagons puts in concrete form notions of equivalent proportional expressions and equivalent fractions, bringing out certain relations between length and area measurement.

F Exploring parallelogram deltagons

Parallelogram deltagons can be defined by an ordered pair of natural numbers (a, b) $a \geq b > 0$, representing the respective length of their two non-parallel sides.

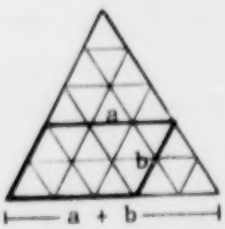


The calculation of parallelogram deltagons can be done in various ways. As we can see, each of these parallelogram deltagons is made of a series of elementary parallelograms each of which is in turn made of 2a elementary triangles.



Since we find a number b of elementary parallelograms within a parallelogram deltagon, the area of the latter will thus be equal to 2ab (elementary triangles).

Any parallelogram deltagon (a, b) can also be seen as what is left from a triangular deltagon (a + b) once we have removed triangles a and b.

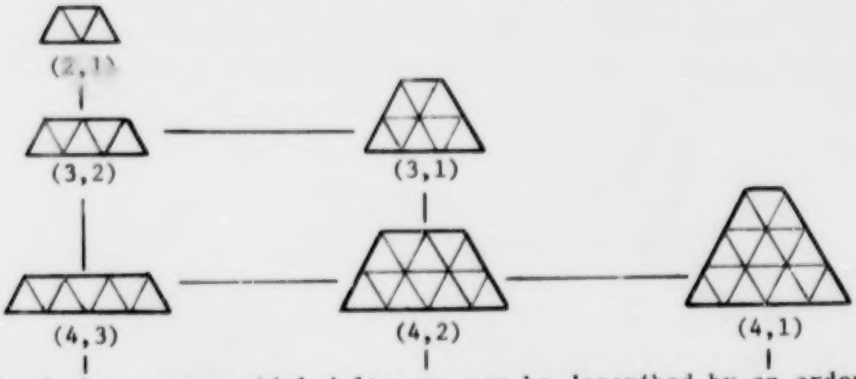


Thus, the area of parallelogram deltagon (a, b) can be obtained by $(a + b)^2 - a^2 - b^2$.

By relating these two ways of calculating the area of parallelogram deltagons, we realize that $2ab = (a + b)^2 - a^2 - b^2$, that is to say: $(a + b)^2 = a^2 + 2ab + b^2$. We have thus found a "deltagonal" representation of the binomial formula.

E Exploring trapezoidal deltagons

First, we will try and find all the possible trapezoidal deltagons:



Each of these trapezoidal deltagons can be described by an ordered pair of natural numbers (p, q), p>q>0, representing the length of the two parallel sides. We can see that the two non-parallel sides are of equal length (p - q). Knowing this, it will be easy to elaborate a formula enabling us to calculate the perimeter of the trapezoidal deltagons.

There are many ways of tackling the problem of calculating the area of trapezoidal deltagons. One of them considers the fact that any trapezoidal deltagon can be seen as what is left of a triangular deltagon p once we have remove the small triangular deltagon q.

Thus, the area of a trapezoidal deltagon (p, q) can be seen as the difference $p^2 - q^2$ between the triangular deltagons p and q.

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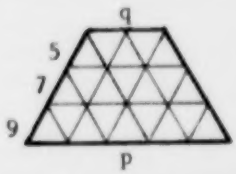

The sum of this succession of odd numbers, from $2q + 1$ to $2p - 1$, will add up to the area of the trapezoidal deltagon.

These two ways of finding the area of a trapezoidal deltagon will bring out certain relations between numbers. For example, the difference between two square numbers can always be expressed by the sum of a series of successive odd numbers; the difference between two successive square numbers is always an odd number, etc.

D Exploring the triangular deltagons

One way of beginning this activity could consist in getting the students to create many different triangular deltagons:

We can see that the only way these deltagons can be differenciated is by their size since they have a similar shape. Each triangular deltagon can be determined by either of the following quantitative factors: p: length of a side, p^2: area (number of elementary triangles of which it is made) and 3p: perimeter.

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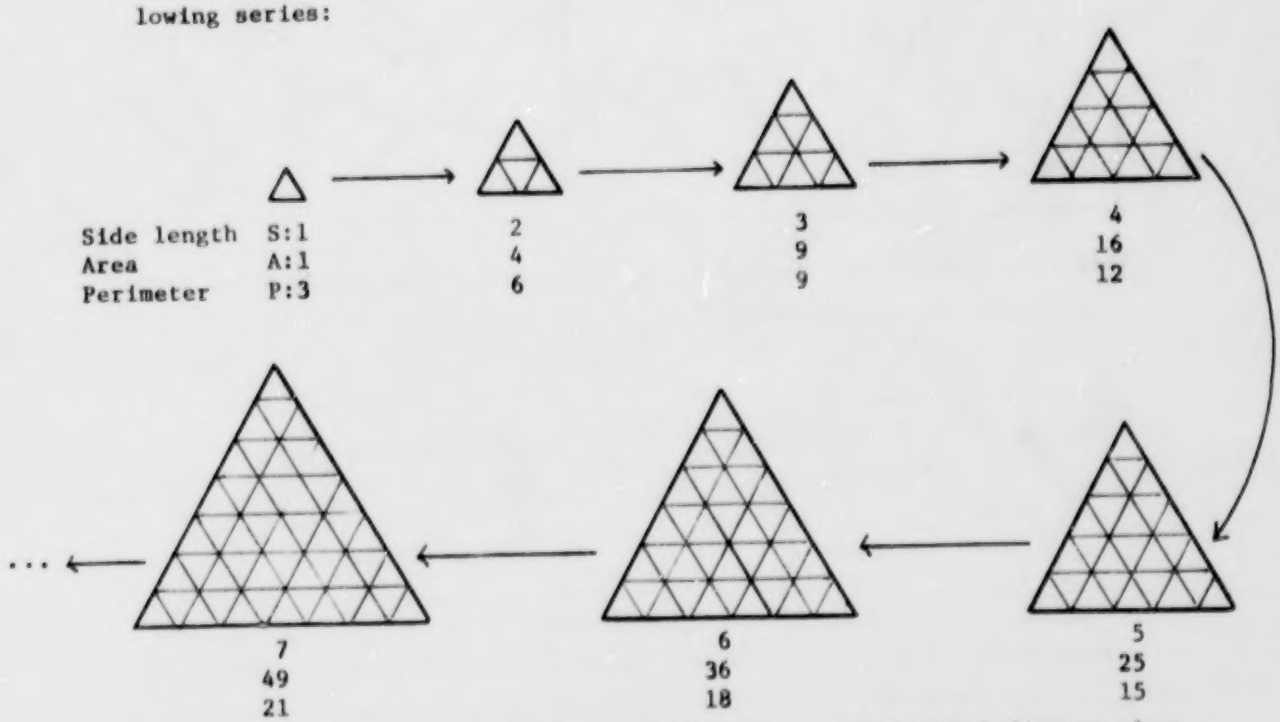

These three series of numbers attached to these geometrical figures show that we can obtain as many triangular deltagons as there are numbers of natural numbers ($p>0$), or square numbers ($p^2>0$), or multiples of three ($3p>0$).

C Deltagon puzzles

Through their activities of composition and decomposition, the puzzles stimulate the learner's spatial imagination and indirectly favor the exploration of geometrical shapes. Such play activities depend on the students' creativity enabling them to compose fanciful or more geometrical shapes. Here are a few examples of this:

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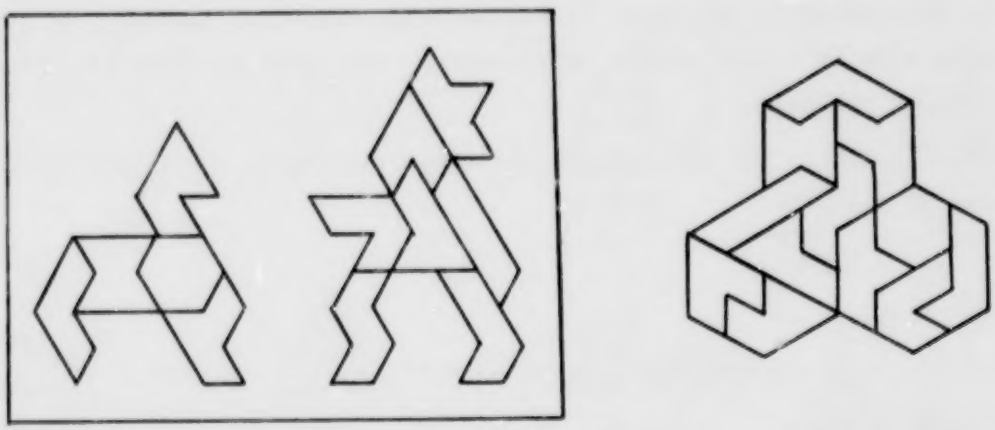
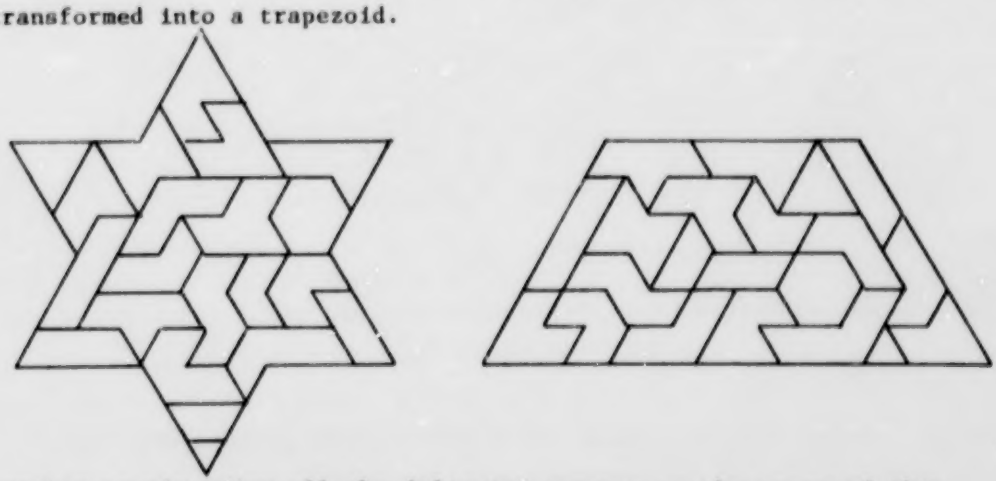


This star made of all the deltagons (except the diamond (◇)) can be transformed into a trapezoid.



Another puzzle using all the deltagons appears on the cover of this brochure.

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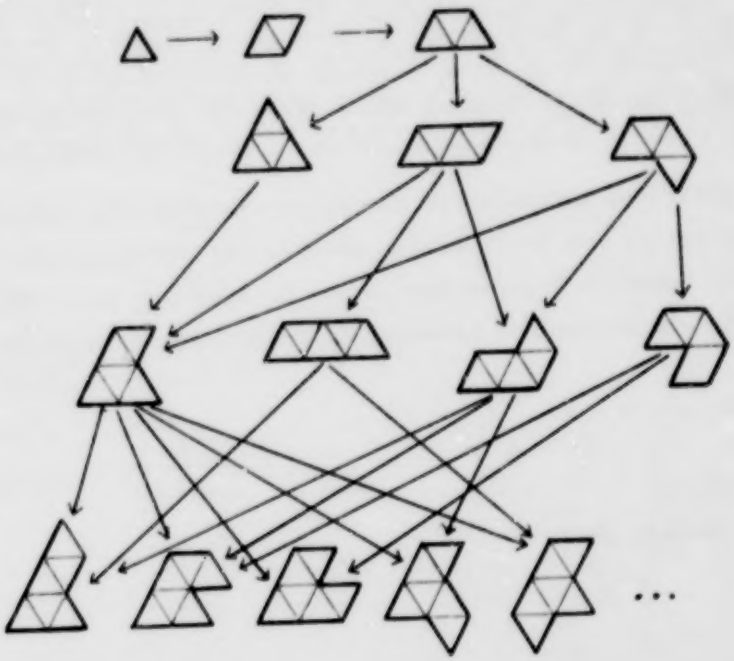
1. Martin Gardner: On polyiamands: shapes that are made of equilateral triangles, Scientific American, Dec. 1964.

B Searching for deltagons made of a specific number of elementary triangles

This activity may be initiated by asking the following question: "How many deltagons made of one, two, three, four, five or six triangles are there?".

While searching for all these deltagons, the students will ponder on a way of forming the various shapes and might discover the need for a more systematic method enabling them to be sure of their results. The elaboration of strategies for the generation of the shapes as well as ways of transforming those already obtained is but one of the skills such an activity can develop. The process illustrated in the following diagram shows the possible ways we can add an elementary triangle to a deltagon.

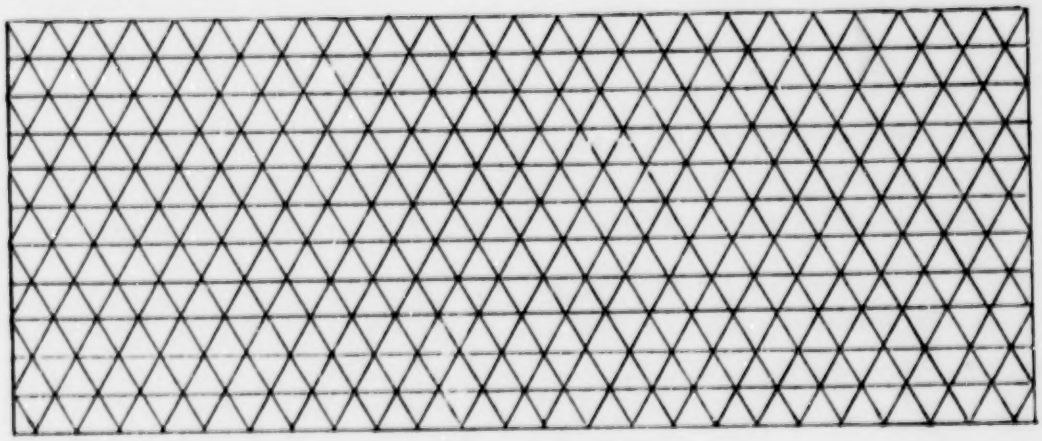


This activity leads the students towards a classification of the deltagons according to their number of elementary triangles, that is, according to their area, their size. It furthers, therefore, the exploration of possible variations of shapes of the same size. All these shapes (of the same size) may afterwards be characterized by other properties such as their number of sides, perimeter, convexity, number of axial symmetries, etc.

POSSIBLE ACTIVITIES

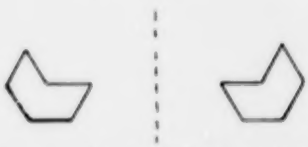
A Creating and discriminating deltagons

By composing equilateral triangles given in the material, we can generate many deltagons the shapes of which we can record by drawing them over the following triangular grid.

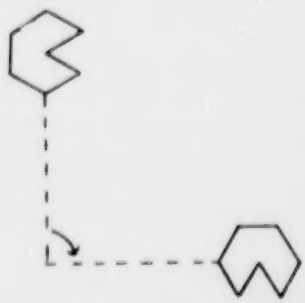


In their effort to generate new deltagons, the students will be brought to ask themselves how to determine whether two deltagons are identical or not. It would be interesting, then, to identify which shapes can be superposed by translation, symmetrical reflection or rotation.

translation



symmetrical reflection

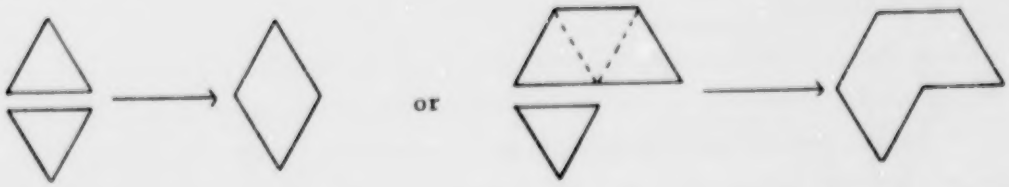


rotation

The description of the various deltagons will lead to the discovery of certain properties of these geometrical shapes such as their number of sides or vertices, their resemblance with familiar geometrical shapes (triangles, trapezoids, parallelograms), the number of elementary triangles of which they are made, their convexity, etc.

DETAILED DESCRIPTION

Equilateral congruent triangles can be joined along their sides generating various polygons.



Since the equilateral triangle is the fundamental element of such polygons, we will call them deltagons (from the shape of the Greek capital letter delta (Δ)).

The polygons thus generated can be characterized by the number of elementary triangles they are made of. In our search for all the possible deltagons that can be composed of one, two, three, four, five and six triangles, we find the following ones:

one triangle:

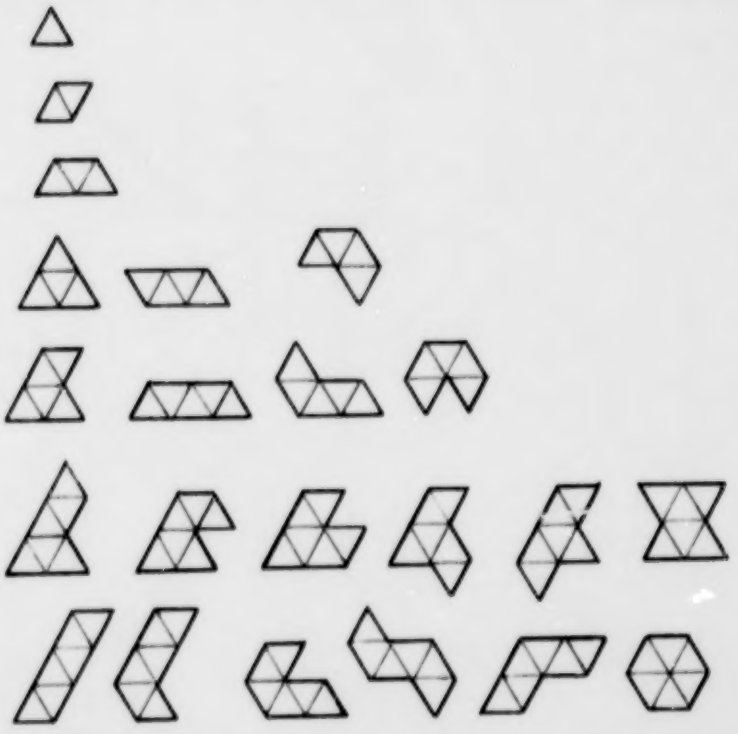
two triangles:

three triangles:

four triangles:

five triangles:

six triangles:



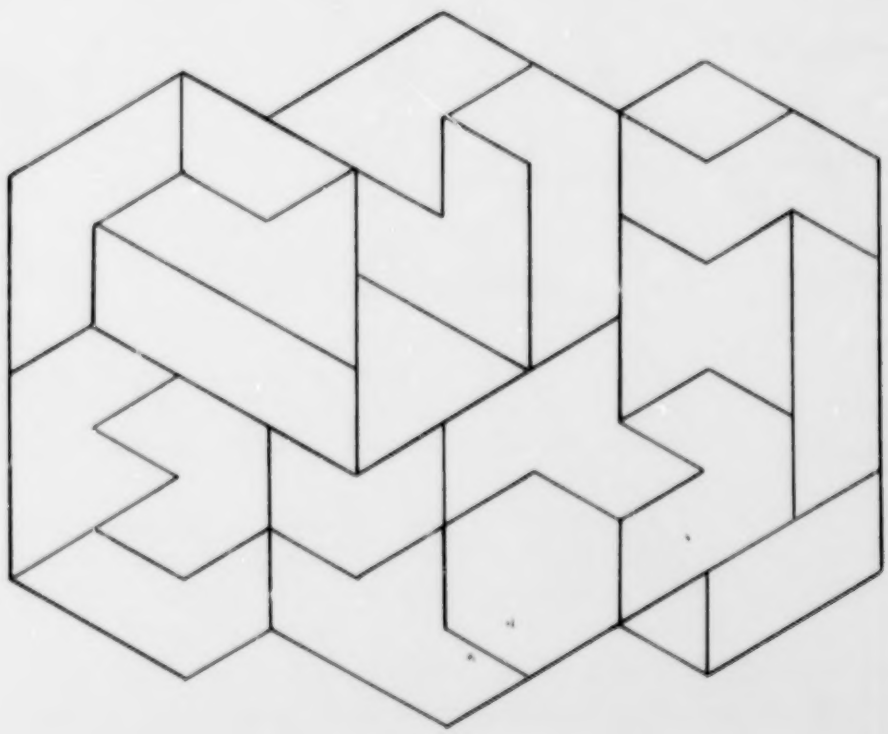
The present material is made of 30 triangles (Δ), 20 diamonds (◇), 10 trapezoids (⏢) and one exemplar deltagons made of four, five and six triangles.

DELTAGONS

GENERAL DESCRIPTION

The following learning material, made of plane geometrical shapes, was conceived to bring concrete support to arithmetical and geometrical activities. There are numerous interrelations between these two fields of mathematics that such a material is able to illustrate. It can bring out unsuspected relations among numbers through the study of certain quantitative properties of deltagons, as well as give a new meaning to already known number facts by putting them into concrete geometrical shapes.

In addition to the quantitative properties of deltagons, one can consider the qualitative properties of these polygonal shapes and also the various ways the shapes may be generated or transformed. The activities we propose emphasize the emergence of processes rather than the acquisition of specific knowledge and, therefore, make room for the student's creativity and personal strategies.



C Exploring similar deltagons

This activity begins by the creation of similar deltagons. Such deltagons may vary in size only. Here is an example:

By observing these deltagons, we see that to be similar, two deltagons must have identical proportions. In other words, sides that are of the same length in one figure must be of equal length in the similar figure; if one side is twice as long as another one in one figure, it must also be the case in the similar figure. Therefore, in the above drawings, we find that the ratio between the basis of the boat and one of its side is 2:1 for the first figure, 4:2 for the second and 6:3 for the third. Different similar deltagons can thus be characterized by proportional expressions that are different but nevertheless equivalent. The relation "same shape" within the realm of geometrical figures corresponds, therefore, to the relation "same proportion" or "same ratio" in the world of numbers.

Similar deltagons can be ordered as equivalent proportional expressions or equivalent fractions according to the size of their constituents. By forming such series of deltagons through the doubling, the tripling, etc., of their corresponding sides, we can observe the effect of those operations on the area. We will then realize that whenever the length of the sides of a deltagon is n-upled, the area of the deltagon is n^2-upled.

This exploration of similar deltagons puts in concrete form notions of equivalent proportional expressions and equivalent fractions, bringing out certain relations between length and area measurement.

REFERENCES

1. Gardner, M.: On polyiamands: shapes that are made of equilateral triangles, Scientific American, December 1964.

2. Lunkenbein, D.: Deltagone: Formen und Zahlen, Mathematiklehrer, 1-1982 und 2-1982.

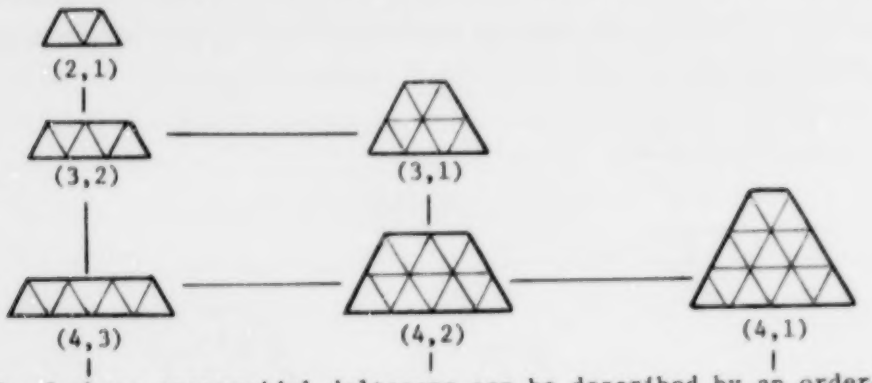
3. Lunkenbein, D.: Deltagons; An Exploration of Shapes and Numbers, Université de Sherbrooke, Département de mathématique et d'informatique, 1982.

Author — D. Lunkenbein
Editor — J. Roy
Drawings (and English translation) — P. Allard

Produced within a research project on the didactic of mathematics subsidized by the Ministère de l'Education du Québec, (F.C.A.C., EQ-646); Département de mathématiques et d'informatique, Université de Sherbrooke, Sherbrooke, Québec, Canada, J1K 2R1.

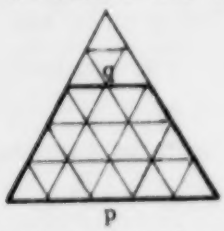
E Exploring trapezoidal deltagons

First, we will try and find all the possible trapezoidal deltagons:



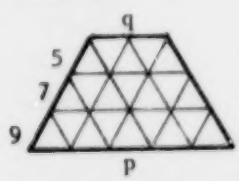
Each of these trapezoidal deltagons can be described by an ordered pair of natural numbers (p, q), p>q>0, representing the length of the two parallel sides. We can see that the two non-parallel sides are of equal length (p - q). Knowing this, it will be easy to elaborate a formula enabling us to calculate the perimeter of the trapezoidal deltagons.

There are many ways of tackling the problem of calculating the area of trapezoidal deltagons. One of them considers the fact that any trapezoidal deltagon can be seen as what is left of a triangular deltagon p once we have remove the small triangular deltagon q.



Thus, the area of a trapezoidal deltagon (p, q) can be seen as the difference $p^2 - q^2$ between the triangular deltagons p and q.

Another way of calculating the area of a trapezoidal deltagon is through the following observation: any trapezoidal deltagon is made of a series of successive elementary trapezoids each of which is made of an odd number of equilateral triangles.



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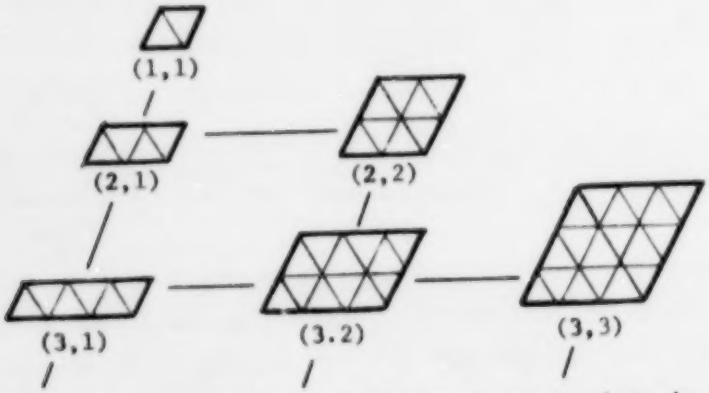
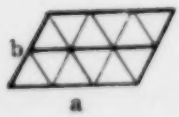
These two ways of finding the area of a trapezoidal deltagon will bring out certain relations between numbers. For example, the difference between two square numbers can always be expressed by the sum of a series of successive odd numbers; the difference between two successive square numbers is always an odd number, etc.

F Exploring parallelogram deltagons

Parallelogram deltagons can be defined by an ordered pair of natural numbers (a, b) $a \geq b > 0$, representing the respective length of their two non-parallel sides.



The calculation of parallelogram deltagons can be done in various ways. As we can see, each of these parallelogram deltagons is made of a series of elementary parallelograms each of which is in turn made of 2a elementary triangles.



Since we find a number b of elementary parallelograms within a parallelogram deltagon, the area of the latter will thus be equal to 2ab (elementary triangles).

Any parallelogram deltagon (a, b) can also be seen as what is left from a triangular deltagon (a + b) once we have removed triangles a and b.

Thus, the area of parallelogram deltagon (a, b) can be obtained by $(a + b)^2 - a^2 - b^2$.

By relating these two ways of calculating the area of parallelogram deltagons, we realize that $2ab = (a + b)^2 - a^2 - b^2$, that is to say: $(a + b)^2 = a^2 + 2ab + b^2$. We have thus found a "deltagonal" representation of the binomial formula.

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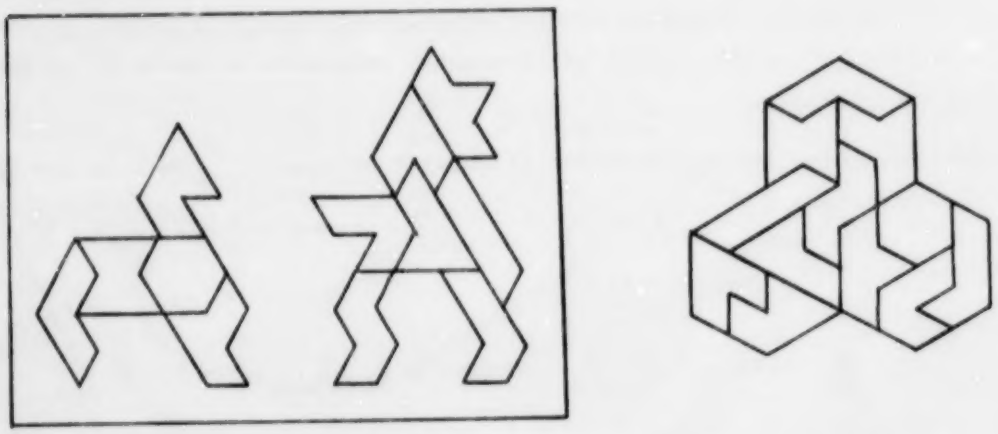

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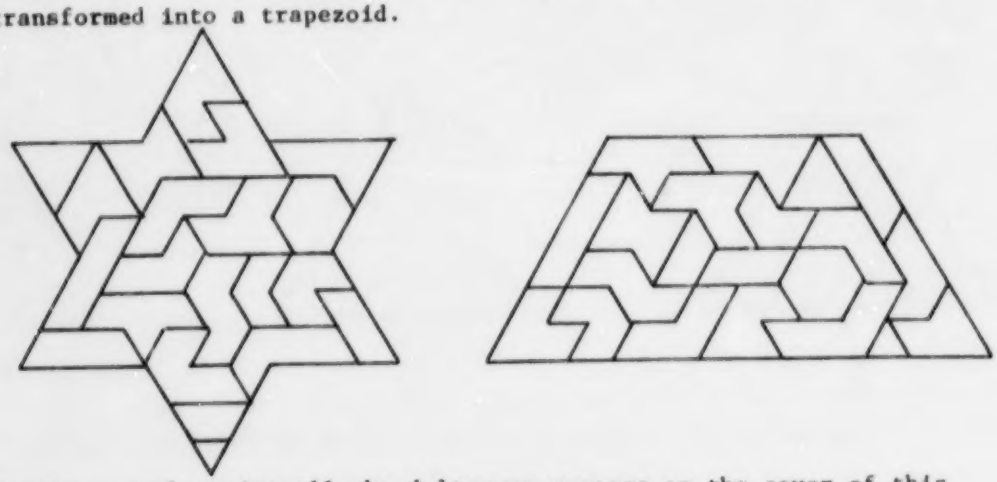

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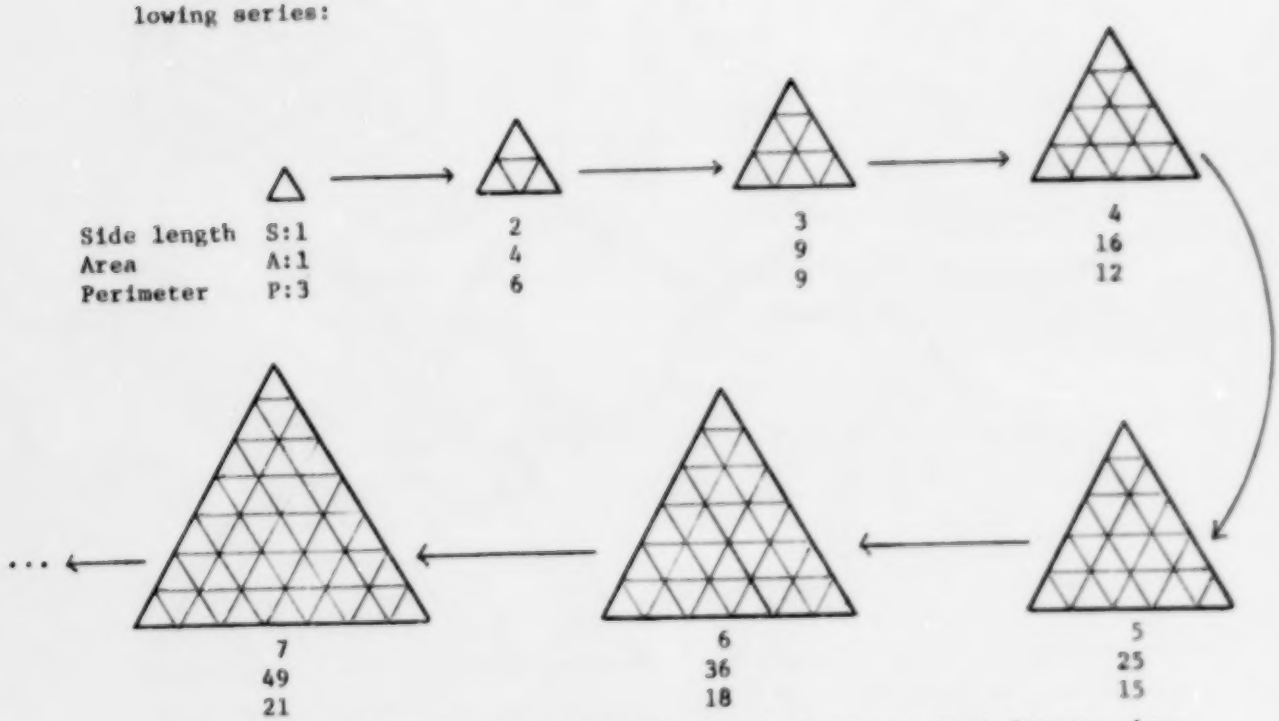

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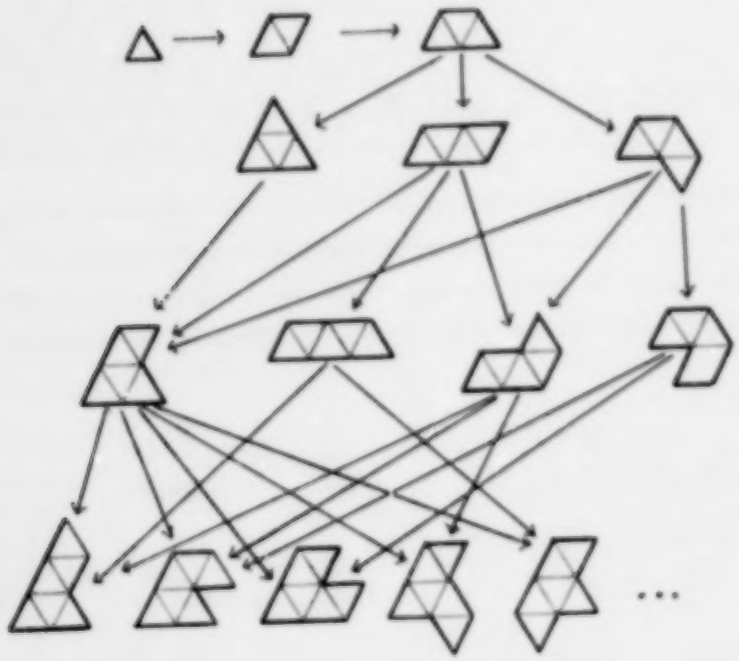

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A Creating and discriminating deltagons

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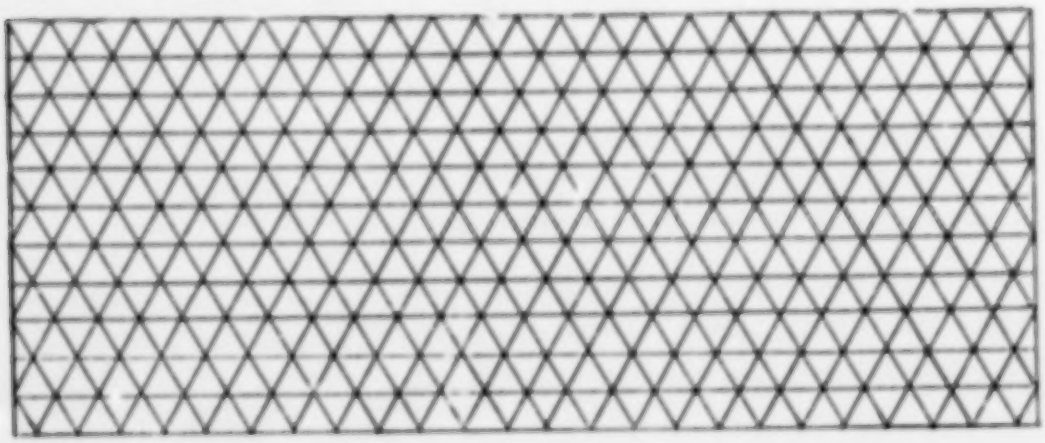
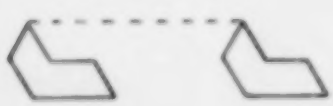
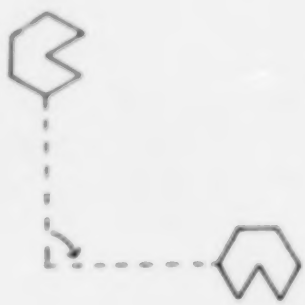


In their effort to generate new deltagons, the students will be brought to ask themselves how to determine whether two deltagons are identical or not. It would be interesting, then, to identify which shapes can be superposed by translation, symmetrical reflection or rotation.



translation

symmetrical reflection



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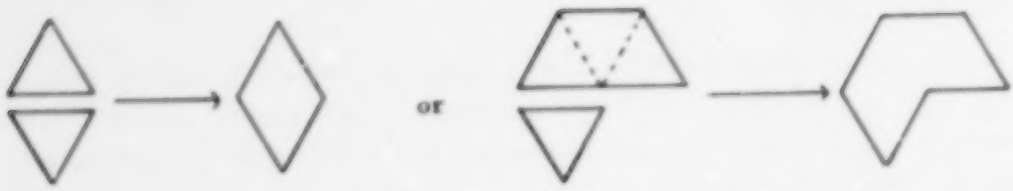

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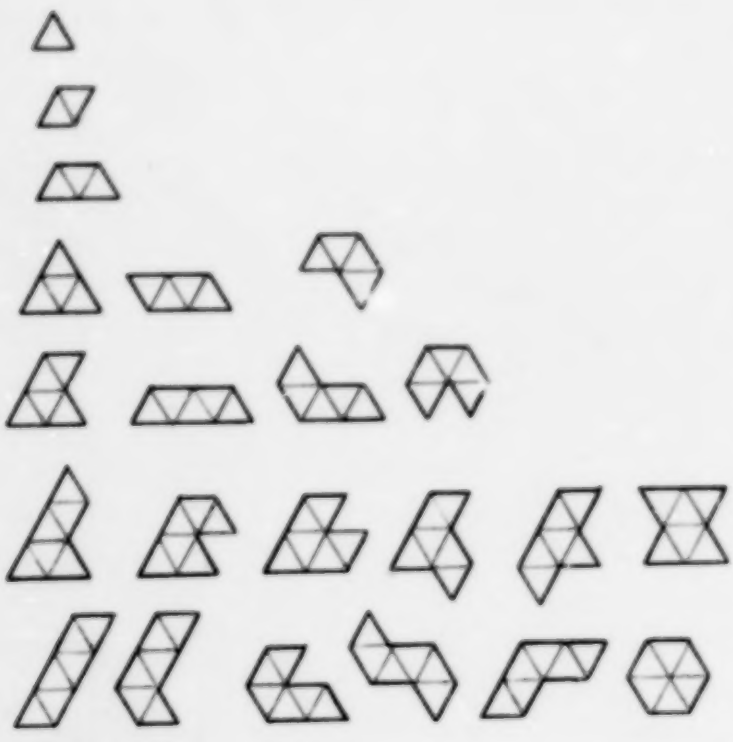

The present material is made of 30 triangles (△), 20 diamonds (▱), 10 trapezoids (⏢) and one exemplar deltagons made of four, five and six triangles.

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